

METODA REPLIKASI PADA SIMULASI SISTEM ANTRIAN M/M/1

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Abstract

Simulasi komputer seringkali sangat ditentukan oleh ukuran sampel yang diambil. Pada umumnya semakin besar ukuran sampel, hasil simulasi menjadi lebih baik. Namun ukuran sampel yang besar membutuhkan waktu dan biaya yang mahal. Ada beberapa metoda yang dapat dipilih untuk menentukan ukuran sampel yang optimum. Pada makalah ini disimpulkan bahwa metoda replikasi yang pada umumnya sering digunakan pada kasus-kasus simulasi komputer yang sederhana seperti suatu antrian M/M/1, ternyata dapat membawa kita kepada kesimpulan/ hasil yang salah (kurang tepat).

Key Word : simulasi, replikasi, sampel

I. LANDASAN TEORI

Hasil suatu simulasi tunggal pada sistem antrian M/M/1 tidak dapat begitu saja dipercaya, sehingga diperlukan suatu interval kepercayaan bagi suatu hasil simulasi dalam hal ini waktu tunggu rata-rata dalam sistem antrian M/M/1.

Pada *metoda replikasi*, jika kita dapat membuat n pengamatan, maka kita bagi n pengamatan tersebut atas k buah replikasi yang saling independen dengan panjang/ukuran sampel masing-masing $m = n/k$. Masing-masing replikasi dimulai dengan kondisi awal yang sama dengan benih bilangan random yang berbeda. Jika $\bar{X}_j(m)$ adalah mean sampel dari m pengamatan yang ke- j , maka $\bar{X}_j(m)$ ($j = 1, 2, \dots, k$) adalah variabel random IID dengan mean $\mu(m) = E[\bar{X}_j(m)]$.

Sebagai estimator bagi μ kita gunakan:

$$\bar{X}(k, m) = \frac{\sum_{j=1}^k \bar{X}_j(m)}{k}$$

dan kita estimasi varian dari $\bar{X}(k,m)$ yaitu $\text{Var}[\bar{X}(k,m)]$ dengan:

$$\hat{\sigma}^2[\bar{X}(k,m)] = \hat{\sigma}^2[k,m] = \frac{\sum_{j=1}^k [\bar{X}_j(m) - \bar{X}(k,m)]^2}{k[k-1]}$$

Jika kita anggap bahwa $\mu(m) = \mu$ dan $\bar{X}_j(m)$ berdistribusi normal maka:

$$\frac{[\bar{X}(k,m) - \mu]}{\hat{\sigma}(k,m)} \dots\dots\dots (1)$$

memiliki *distribusi t* dengan derajat kebebasan $k-1$. Dengan demikian 90% interval kepercayaan bagi μ adalah:

$$\bar{X}(k,m) \pm t_{k-1,0.95} \hat{\sigma}(k,m) \dots\dots\dots(2)$$

Namun, $\mu(m)$ umumnya berbeda dari μ (oleh karena efek keadaan awal sistem yang disimulasikan, sehingga $\bar{X}(k,m)$ merupakan estimator yang bias bagi μ). Lagi pula $\bar{X}_j(m)$ ($1,2,\dots,k$) umumnya tidak berdistribusi normal untuk k yang tidak terlampau besar.

Oleh sebab itu, rumus (1) tidak benar-benar berdistribusi-t dan akibatnya interval kepercayaan yang dibentuk menurut rumus (2) belum tentu memiliki cakupan sebesar 90% seperti yang kita inginkan.

Law [7] menemukan bahwa kesalahan yang paling besar disebabkan oleh kenyataan bahwa $\mu(m) \neq \mu$. Jika m ditetapkan terlalu kecil, maka $\mu(m)$ berbeda besar dari μ . Bila k dan juga n membesar, maka cakupan dari 90% interval kepercayaan yang terbentuk dapat sangat rendah. Ini disebabkan karena untuk m yang tetap, bertambahnya k menyebabkan bertambah pendeknya selang interval kepercayaan bagi $\mu(m)$, bukan untuk μ , sehingga interval kepercayaan ini semakin sedikit mengandung μ .

II. HASIL PERCOBAAN

Untuk sistem antrian M/M/1, kita peroleh hasil seperti pada *tabel-1* berikut. Pada tabel tersebut ditunjukkan suatu hasil metoda replikasi dengan banyak replikasi $k = 40$ dan ukuran sampel m yang berbeda-beda, berkisar dari 5 sampai dengan 4000.

Pengambilan banyak replikasi sebesar 40 buah dimaksudkan agar $\bar{X}_i(m)$ ($i = 1, 2, \dots, 40$) dapat dianggap berdistribusi normal (menurut *dalil limit sentral* dalam statistik). Dari tabel-1 serta grafik tersebut tampak bahwa semakin besar ukuran m , untuk banyak replikasi (k) yang tetap (dengan demikian banyak pengamatan n bertambah besar), maka hasil yang diperoleh yaitu waktu tunggu rata-rata dalam antrian yang telah di rata-ratakan 40 kali ($\bar{X}(m)$) berfluktuasi dengan simpangan/varian yang kecil di sekitar nilai analitiknya yaitu $d = 0.5$. Ini *mungkin* dapat diartikan bahwa ukuran sampel kita sudah cukup besar untuk mengestimasi waktu tunggu rata-rata sistem yang sebenarnya.

Sekarang perhatikanlah replikasi untuk $m = 50$ dan $m = 90$, yang memberikan hasil $\bar{X}(50) = 0.5252$ dan $\bar{X}(90) = 0.4998395$. Hasil ini ternyata mendekati hasil analitiknya sekalipun ukuran sampel m tidak terlalu besar. Namun perlu diingat bahwa masing-masing hasil ini diperoleh dari satu kali simulasi dengan replikasi sebanyak 40 buah. Bagaimanakah cakupan sebenarnya dari 90% interval kepercayaan yang dibentuk dengan metoda replikasi dengan banyak replikasi 40 buah dan ukuran sampel 50 dan 90 ini?

SISTEM ANTRIAN M/M/1 DENGAN $\rho = -5$

Waktu tunggu rata-rata analitik = 0,5

Untuk tiap banyak pengamatan simulasi dilakukan = 40 kali

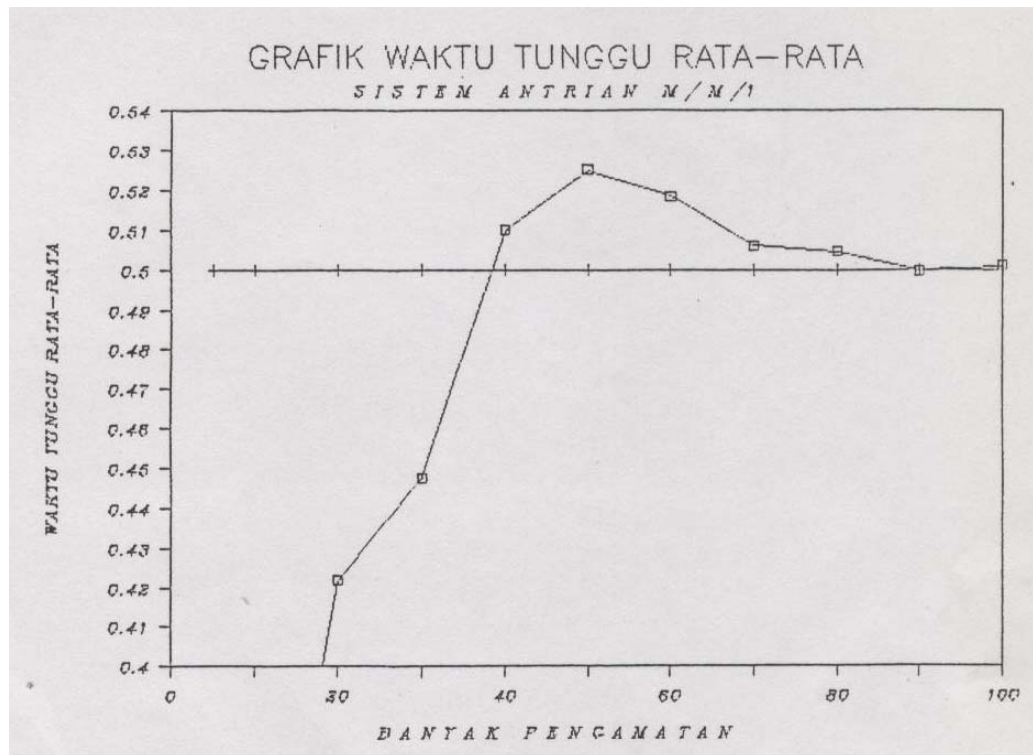
BANYAK PENGAMATAN	WAKTU TUNGGU RATA-RATA			STANDAR DEVIASI	90% INTERVAL KEPERCAYAAN
	TERKECIL	TERBESAR	RATA-RATA		
5	.0000000	1.3128109	.2320367	.0463141	[.1534417 , .3106316]
10	.0000000	1.2006843	.3027324	.0396175	[.2355014 , .3699634]
20	.0390218	1.3146975	.4217791	.0534606	[.3310564 , .5125018]
30	.0681481	1.7898959	.4476196	.0529937	[.3576893 , .5375500]
40	.0968305	1.5849271	.5102128	.0511827	[.4233557 , .5970699]
50	.1145122	1.5847347	.5252000	.0559096	[.4303213 , .6200786]
60	.1239559	1.7896181	.5185806	.0562749	[.4230821 , .6140792]
70	.1183758	1.5945373	.5062680	.0490843	[.4229720 , .5895641]
80	.1138549	1.4171146	.5046036	.0440230	[.4298966 , .5793105]
90	.1201159	1.2917405	.4998395	.0403342	[.4313924 , .5682865]
100	.1354063	1.1799703	.5011013	.0376677	[.4371791 , .5650234]
150	.1788087	1.1671364	.5382505	.0349740	[.4788995 , .5976014]
200	.2734459	1.0601109	.5179032	.0295924	[.4676850 , .5681214]
250	.2326723	.9503601	.5163580	.0265183	[.4713564 , .5613596]
300	.3204374	.8830336	.5179877	.0226466	[.4795564 , .5564190]
350	.3051660	.9117878	.5041972	.0217958	[.4672097 , .5411848]
400	.2963198	.8447024	.4927015	.0203436	[.4581784 , .5272245]
450	.2870188	.7839468	.4870938	.0180116	[.4565280 , .5176595]
500	.3105778	.7392131	.4960830	.0168833	[.4674321 , .5247339]
550	.3296686	.7213785	.5000407	.0157850	[.4732536 , .5268278]
600	.3325157	.7176896	.5010093	.0145275	[.4763561 , .5256625]
650	.3240494	.6828105	.5032328	.0136570	[.4800569 , .5264088]
700	.3216453	.6794062	.4978992	.0132393	[.4754321 , .5203663]
750	.3152023	.6489068	.4942060	.0130320	[.4720906 , .5163213]
800	.3266279	.6750981	.4926456	.0126555	[.4711693 , .5141219]
850	.3332206	.6766452	.4946535	.0133678	[.4719684 , .5173386]
900	.3394862	.6537350	.4940429	.0129752	[.4720240 , .5160618]
950	.3458884	.6524202	.4939282	.0122820	[.4730856 , .5147708]
1000	.3499532	.6312147	.4938389	.0119635	[.4735369 , .5141409]
1100	.3581162	.6451380	.4925627	.0109865	[.4739186 , .5112068]
1200	.3818677	.6674779	.5000526	.0118636	[.4799201 , .5201851]
1300	.3774429	.6671612	.5022301	.0115000	[.4827146 , .5217455]
1400	.3823792	.6454426	.4990337	.0109214	[.4805000 , .5175674]
1500	.3875671	.6390870	.4992576	.0109857	[.4806148 , .5179003]
1600	.3772194	.6359589	.4985053	.0107313	[.4802943 , .5167163]
1700	.3716752	.6284684	.4980019	.0106865	[.4798668 , .5161369]
1800	.3728733	.6113597	.4945556	.0100518	[.4774977 , .5116134]

Table-1

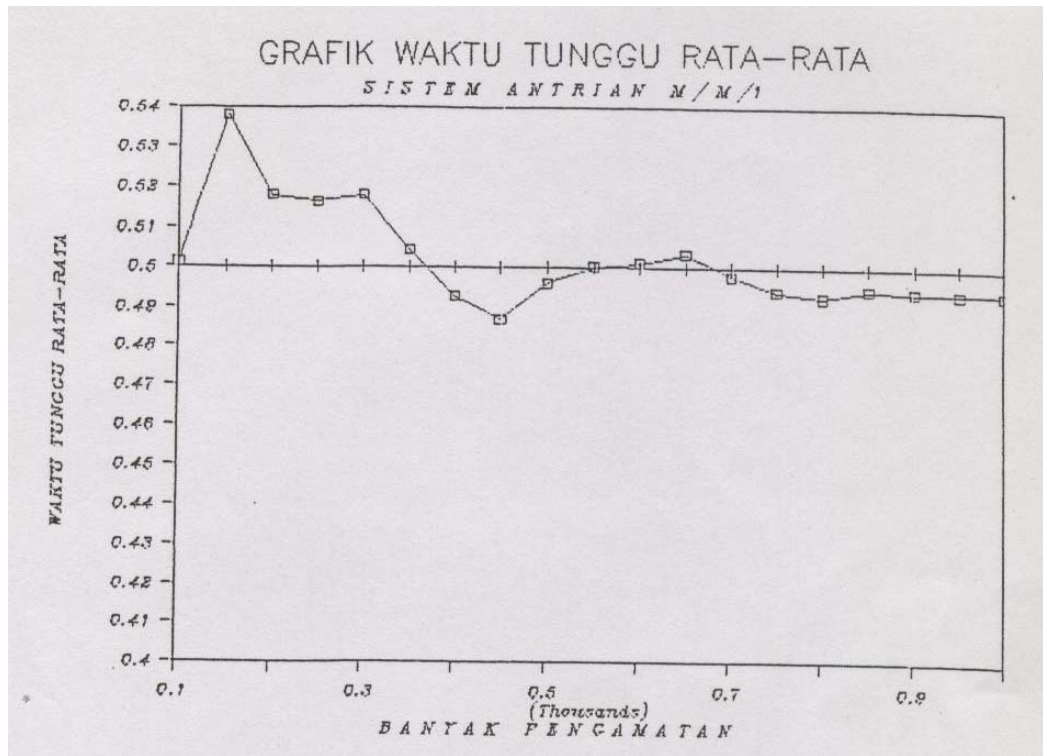
Untuk tiap banyak pengamatan simulasi dilakukan = 40 kali

BANYAK PENGAMATAN	WAKTU TUNGGU RATA-RATA			STANDAR DEVIASI	90% INTERVAL KEPERCAYAAN
	TERKECIL	TERBESAR	RATA-RATA		
1900	.3690845	.6170863	.4953613	.0096841	[.4789274 , .5117652]
2000	.3676870	.6288874	.4933281	.0093647	[.4774362 , .5092200]
2100	.3763914	.6209010	.4933570	.0093788	[.4774413 , .5092728]
2200	.3780612	.6065154	.4930248	.0087728	[.4781375 , .5079122]
2300	.3784792	.6081135	.4930361	.0083052	[.4789422 , .5071300]
2400	.3828709	.5958868	.4939943	.0079704	[.4804685 , .5075201]
2500	.3812266	.5812115	.4952115	.0081601	[.4813638 , .5090591]
2600	.3851071	.5788924	.4945654	.0081985	[.4806525 , .5084783]
2700	.3854793	.5803664	.4929503	.0076853	[.4799083 , .5059922]
2800	.3877638	.5700988	.4923452	.0074575	[.4796898 , .5050005]
2900	.3832905	.5627393	.4921223	.0077209	[.4790198 , .5052247]
3000	.3794924	.5686610	.4911552	.0077906	[.4779345 , .5043759]
3100	.3823803	.5591025	.4907356	.0075945	[.4778477 , .5036235]
3200	.3886458	.5532242	.4914376	.0075316	[.4786565 , .5042186]
3336	.3827040	.5477544	.4889379	.0075685	[.4760941 , .5017816]
3448	.3867404	.5544177	.4902076	.0079070	[.4767895 , .5036258]
3600	.3907380	.5677096	.4927020	.0080978	[.4789600 , .5064440]
3984	.4029350	.5784918	.4915520	.0077763	[.4783556 , .5047483]
4000	.4029828	.5773248	.4912756	.0077557	[.4781143 , .5044370]

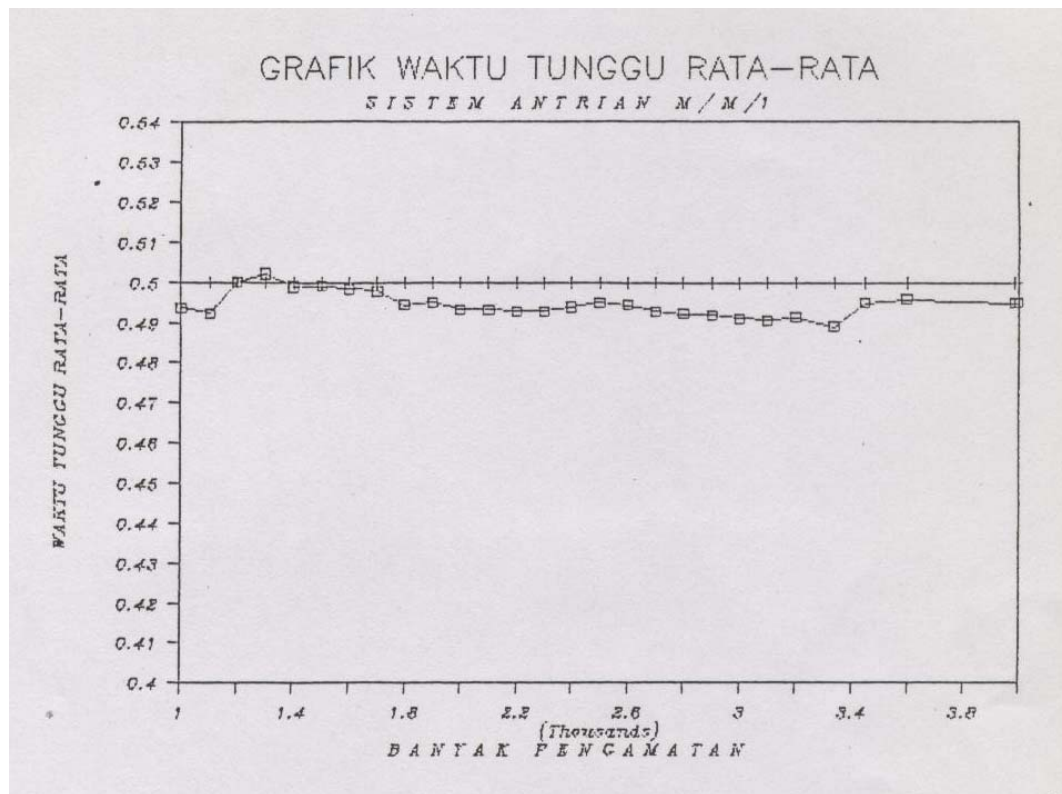
Lanjutan Tabel-1



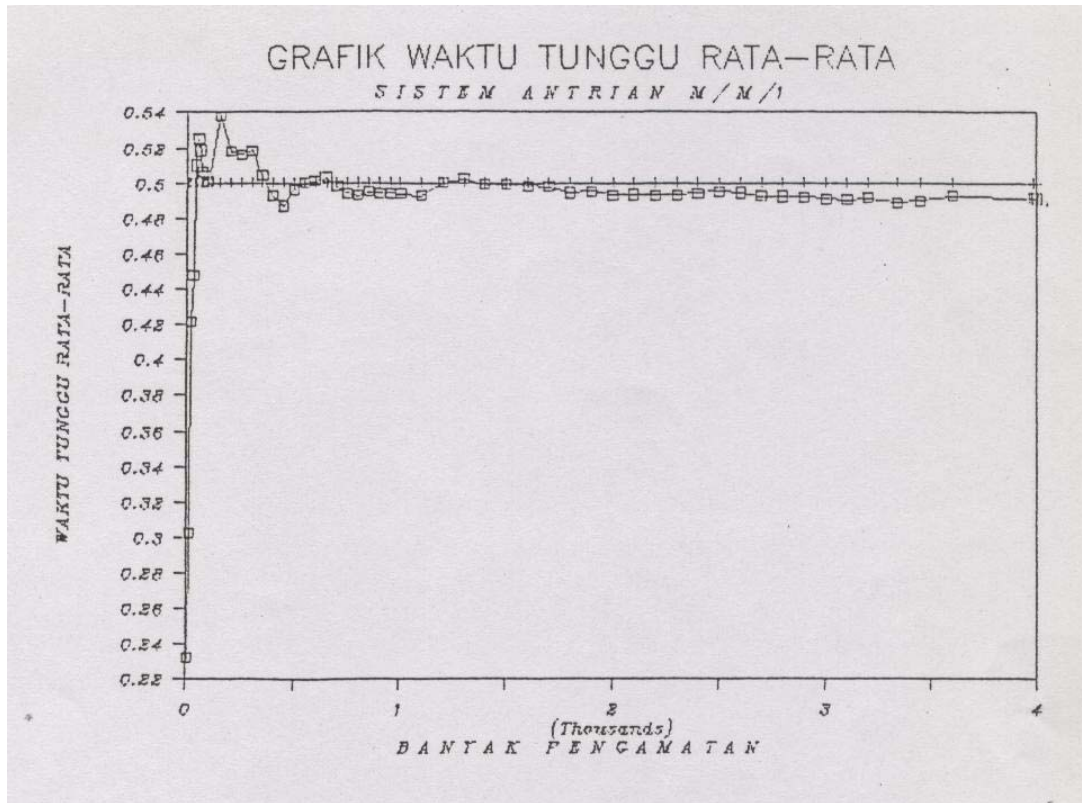
Grafik-1a



Grafik-1b



Grafik-1c



Grafik-1d
(Gabungan dari Grafik-1a, 1b dan 1c)

Untuk mengetahuinya, kita lakukan 100 kali percobaan simulasi dengan metoda replikasi dengan tiap simulasi memiliki banyak replikasi = 40 dan ukuran sampel tiap replikasi diambil 50 dan 90 buah.

Hasil ini dapat kita lihat pada tabel-2 dan tabel-3. Ternyata dari 100 buah 90% interval kepercayaan yang dibentuk, diperoleh:

Banyak replikasi	Ukuran sampel	90% interval kepercayaan bagi \hat{p}
40	50	0.65 ± 0.08
40	90	0.72 ± 0.07

90% interval kepercayaan bagi \hat{p} ini dapat dihitung dengan rumus:

$$\hat{p} \pm 1.645 \sqrt{\frac{\hat{p}(1-\hat{p})}{100}}$$

dengan \hat{p} =proporsi banyak interval yang mengandung $d=0.5$ pada tabel 2 dan tabel 3.

Ternyata 90% interval kepercayaan yang dibentuk bagi \hat{p} dengan metoda replikasi ini (untuk banyak replikasi = 40 dan ukuran sampel 50 serta 90) tidak mengandung 0.9 (90%), seperti yang kita inginkan.

METODA REPLIKASI

Banyak replikasi tiap simulasi = 40

Banyak pengamatan tiap replikasi = 50

SIMULASI KE	90% INTERVAL KEPERCAYAAN	WAKTU TUNGGU RATA-RATA	PRESISI RELATIF
1	[.4601693, .6295627]	.544866	.1554451
2	[.3921402, .609394]	.5007671	.216921
3	[.4064503, .6284147]	.5174325	.2144863
4	[.3908756, .5229272]	.4569014	.1445077
5	[.3521139, .4939761]	.423045	.1676679
6	[.3330761, .5024076]	.4177418	.2026747
7	[.3015046, .4456697]	.3735871	.1929471
8	[.3644997, .4829502]	.423725	.1397728
9	[.369848 , .5469054]	.4583767	.1931352
10	[.3896404, .5467974]	.4682189	.1678243
11	[.4601693, .6295627]	.544866	.1554451
12	[.3921402, .609394]	.5007671	.216921
13	[.4064503, .6284147]	.5174325	.2144863
14	[.3566559, .4771469]	.4169014	.1445077
15	[.3521139, .4939761]	.423045	.1676679
16	[.3330761, .5024076]	.4177418	.2026747
17	[.3015046, .4456697]	.3735871	.1929471
18	[.36456 , .4830301]	.4237951	.1397728
19	[.3698481, .5469053]	.4583767	.1931352
20	[.3896404, .5467974]	.4682189	.1678243
21	[.4601693, .6295627]	.544866	.1554451
22	[.3921402, .609394]	.5007671	.216921
23	[.4064503, .6284147]	.5174325	.2144863
24	[.3729102, .4988925]	.4359014	.1445077
25	[.3521139, .4939761]	.4230453	.1676679
26	[.3330761, .5024076]	.4177418	.2026747
27	[.3015046, .4456697]	.3735871	.1929471
28	[.3731623, .4944278]	.4337951	.1397728
29	[.369848 , .5469054]	.4583767	.1931352
30	[.3896404, .5467974]	.4682189	.1678243
31	[.3580158, .5127301]	.435373	.1776802
32	[.4225937, .612662]	.5176278	.1835956
33	[.3839215, .5403568]	.4621391	.1692512
34	[.4125409, .5952783]	.5039096	.1813196
35	[.3590161, .4777087]	.4183624	.1418538
36	[.4236641, .5738256]	.4987448	.1505394
37	[.3675859, .4990841]	.433335	.151728
38	[.3402777, .4482083]	.394243	.1368834
39	[.3552328, .5003672]	.4278	.1696288
40	[.4089893, .5879977]	.4984935	.1795493
41	[.319691 , .4369873]	.3783391	.1550147
42	[.3642938, .4924634]	.4283786	.1495985

Tabel-2

Banyak replikasi tiap simulasi = 40

Banyak pengamatan tiap replikasi = 50

SIMULASI KE	90% INTERVAL KEPERCAYAAN	WAKTU TUNGGU RATA-RATA	PRESISI RELATIF
43	[.3755001, .6192378]	.497369	.2450271
44	[.3840399, .6105968]	.4973183	.2277786
45	[.4067595, .5954698]	.5011147	.1882906
46	[.3199592, .4760819]	.3980205	.1961238
47	[.3494871, .5238028]	.436645	.1996081
48	[.3317808, .4590764]	.3954286	.160959
49	[.3281272, .4516553]	.3898912	.1584136
50	[.3480105, .4818847]	.4149476	.1613145
51	[.3677271, .5204697]	.4440984	.1719693
52	[.3480207, .4437511]	.3958859	.1209065
53	[.3866843, .5566546]	.4716694	.1801795
54	[.419442, .5943993]	.5069206	.1725687
55	[.3307059, .4868988]	.4088023	.1910372
56	[.4064639, .6059489]	.5062064	.1970392
57	[.3903267, .5294092]	.459868	.15122
58	[.2869261, .3800737]	.3334999	.1396516
59	[.3496758, .5116327]	.4306542	.1880358
60	[.3558689, .5041381]	.4300035	.1724046
61	[.3780737, .5313956]	.4547347	.1685839
62	[.3676012, .5264871]	.4470441	.1777072
63	[.3640937, .5094084]	.436751	.1663588
64	[.350521, .4859657]	.4182434	.1619209
65	[.3774694, .5177305]	.4476	.1566812
66	[.3797579, .6320869]	.5059224	.2493752
67	[.3295575, .5047716]	.4171645	.210006
68	[.3708812, .5314963]	.4511887	.1779911
69	[.3479322, .5353654]	.4416488	.2121971
70	[.3735408, .5369774]	.4552591	.1794984
71	[.3168837, .4447451]	.3808144	.167879
72	[.371818, .499942]	.43588	.146971
73	[.3371777, .4983912]	.4177845	.1929386
74	[.4761364, .6944692]	.5853028	.1865127
75	[.3591644, .5101532]	.4346588	.1736866
76	[.3816597, .5214567]	.4515582	.1547939
77	[.3625056, .4741041]	.4183049	.1333938
78	[.4302024, .6427105]	.5364564	.1980664
79	[.3740729, .5293273]	.4517001	.1718556
80	[.3582062, .5430736]	.4506399	.2051166
81	[.3488071, .4709674]	.4098872	.149017
82	[.417403, .546034]	.4817185	.1335127
83	[.33327, .4573905]	.3953303	.1569833
84	[.3637147, .5319096]	.4478122	.1877962
85	[.3891541, .5366607]	.4629074	.1593262
86	[.4209899, .6742149]	.5476024	.2312125

Lanjutan tabel-2

Banyak replikasi tiap simulasi = 40

Banyak pengamatan tiap replikasi = 50

SIMULASI KE	90% INTERVAL KEPERCAYAAN	WAKTU TUNGGU RATA-RATA	PRESISI RELATIF
87	[.3422706, .4470699]	.3946703	.1327682
88	[.3790515, .6578126]	.518432	.2688503
89	[.4112135, .5467676]	.4789906	.1414997
90	[.29997, .3940701]	.3470201	.1355831
91	[.3672682, .5397659]	.4535171	.1901777
92	[.3499869, .5172235]	.4336052	.1928443
93	[.3476183, .4766831]	.4121507	.1565747
94	[.4874165, .6633353]	.5753759	.152873
95	[.441467, .7435575]	.5925123	.2549234
96	[.381667, .6685251]	.5250961	.2731482
97	[.3040627, .414284]	.3591734	.1534375
98	[.3136193, .4895726]	.401596	.2190676
99	[.4289276, .5989438]	.5139357	.1654061
100	[.3925858, .6029282]	.497757	.2112902

Lanjutan tabel-2

Proporsi banyak interval yang mengandung waktu tunggu rata-rata yang dihitung secara analitik ($=0.5$) adalah : 65%

90% interval kepercayaan bagi proporsi tersebut : 0.65 ± 0.08

Nilai presisi relatif rata-rata adalah : .1781314

METODA REPLIKASI

Banyak replikasi tiap simulasi = 40

Banyak pengamatan tiap replikasi = 90

SIMULASI KE	90% INTERVAL KEPERCAYAAN	WAKTU TUNGGU RATA-RATA	PRESISI RELATIF
1	[.4170309, .5029348]	.4599829	.0933773
2	[.3811607, .5033962]	.4422784	.1381884
3	[.3549887, .4546772]	.404833	.123122
4	[.293444, .440766]	.367105	.200653
5	[.4481834, .6360676]	.5421255	.1732848
6	[.401012, .5220422]	.4615271	.1311194
7	[.442705, .5428494]	.4927772	.1016122
8	[.4336388, .5970395]	.5153391	.1585371
9	[.4166529, .6260008]	.5213268	.2007837
10	[.4165727, .5629045]	.4897386	.1493979
11	[.4087968, .5243704]	.4665836	.123851
12	[.4281451, .6061302]	.5171377	.1720867
13	[.4157912, .5512403]	.4835157	.1400668
14	[.3957666, .5073691]	.4515678	.1235723
15	[.4229075, .5575194]	.4902135	.1372992
16	[.4414666, .5999407]	.5207037	.152173
17	[.4393339, .6164874]	.5279132	.1677817
18	[.3926155, .52436]	.4584878	.1436729
19	[.4241142, .5645119]	.4943131	.142013
20	[.4109846, .5481061]	.4795454	.1429704
21	[.3356723, .4455634]	.3906179	.1406631
22	[.3973597, .4956431]	.4465014	.1100594
23	[.3773968, .5023303]	.4398635	.142014
24	[.370219, .4964443]	.4333316	.1456451
25	[.3705202, .4765082]	.4235142	.1251292
26	[.4339531, .5660483]	.5000007	.132095
27	[.3950177, .5122797]	.4536487	.1292431
28	[.3650201, .4601058]	.4125629	.1152378
29	[.4385809, .6258824]	.5322316	.1759586
30	[.3624108, .4575211]	.4099659	.1159978
31	[.4117331, .5843412]	.4980372	.1732883
32	[.4314583, .5367546]	.4841065	.1087532
33	[.4052857, .5470018]	.4761438	.1488165
34	[.4359788, .5660021]	.5009905	.1297662
35	[.4542354, .6069402]	.5305878	.1439016
36	[.4776959, .6339177]	.5558068	.1405361
37	[.4555594, .6615455]	.5585525	.1843929
38	[.3600746, .4721654]	.41612	.1346856
39	[.4098264, .6252186]	.5175225	.2080994
40	[.4064263, .5458636]	.476145	.1464233
41	[.3879399, .4900686]	.4390043	.1163187
42	[.4223357, .620149]	.5212424	.1897517

Tabel-3

Banyak replikasi tiap simulasi = 40

Banyak pengamatan tiap replikasi = 90

SIMULASI KE	90% INTERVAL KEPERCAYAAN	WAKTU TUNGGU RATA-RATA	PRESISI RELATIF
43	[.4164358, .5308293]	.4736325	.1207618
44	[.3862858, .502434]	.4443599	.1306915
45	[.4199401, .5704106]	.4951754	.1519366
46	[.4375479, .652935]	.5452415	.1975153
47	[.4080316, .5490599]	.4785458	.147351
48	[.4252247, .5392743]	.4822495	.1182475
49	[.3806909, .5027494]	.4417202	.1381627
50	[.3972139, .5195978]	.4584059	.1334886
51	[.3859419, .4834382]	.4346901	.1121446
52	[.4445868, .8456258]	.6451063	.3108317
53	[.4536759, .6271021]	.540389	.1604643
54	[.3672535, .4779345]	.422594	.1309543
55	[.3994604, .5388129]	.4691366	.1485201
56	[.4179909, .5825059]	.5002484	.1644333
57	[.4866441, .723261]	.6049526	.1955665
58	[.4524757, .5994958]	.5259857	.1397567
59	[.3873384, .5380649]	.4627017	.1628766
60	[.465928, .6141849]	.5400565	.1372605
61	[.4379749, .5903415]	.5141582	.148171
62	[.379443, .4831548]	.4312989	.120232
63	[.383548, .4716264]	.4275872	.1029946
64	[.3749965, .4657016]	.420349	.1078926
65	[.3906012, .4833079]	.4369546	.1060827
66	[.3688748, .4822007]	.4255378	.1331561
67	[.3899017, .4777731]	.4338374	.1012723
68	[.3785647, .4718232]	.425194	.1096658
69	[.3616524, .4779175]	.419785	.1384818
70	[.4729805, .6417606]	.5573705	.1514074
71	[.417784, .5418134]	.4797987	.1292515
72	[.3800677, .5005944]	.4403311	.1368593
73	[.4333673, .6887559]	.5610616	.2275941
74	[.4007606, .5678089]	.4842847	.1724691
75	[.4244674, .5563811]	.4904242	.1344894
76	[.3927887, .4846407]	.4387147	.1046831
77	[.383121, .4876291]	.4353751	.1200207
78	[.4090973, .573107]	.4911021	.1669813
79	[.3901528, .5061374]	.4481451	.1294052
80	[.3943762, .5094737]	.451925	.1273414
81	[.383636, .480291]	.4319635	.1118786
82	[.4387139, .5530707]	.4958923	.1153041
83	[.4477772, .5822428]	.51501	.1305465
84	[.4141538, .5639064]	.4890301	.1531118
85	[.4028129, .5252488]	.4640308	.1319264
86	[.4649924, .6153519]	.5401722	.1391774

Lanjutan tabel-3

Banyak replikasi tiap simulasi = 40

Banyak pengamatan tiap replikasi = 90

SIMULASI KE	90% INTERVAL KEPERCAYAAN	WAKTU TUNGGU RATA-RATA	PRESISI RELATIF
87	[.4223328, .5694383]	.4958856	.1483261
88	[.4128199, .5327866]	.4728033	.1268675
89	[.4793208, .6109813]	.5451511	.1207559
90	[.3325629, .4538391]	.393201	.1542165
91	[.3185924, .4426616]	.380627	.162979
92	[.5030741, .6933445]	.5982093	.1590333
93	[.4246052, .61209]	.5183476	.1808485
94	[.3638281, .4564998]	.410164	.1129691
95	[.4578564, .5849996]	.521428	.1219183
96	[.4426979, .5630868]	.5028923	.1196965
97	[.3338089, .4505311]	.39217	.1488158
98	[.4293156, .5613064]	.495311	.1332403
99	[.3941476, .5103328]	.4522402	.1284552
100	[.3633450, .4769830]	.420164	.1352305

Lanjutan tabel-3

Proporsi banyak interval yang mengandung waktu tunggu rata-rata yang dihitung secara analitik ($=0.5$) adalah : 72%

90% interval kepercayaan bagi proporsi tersebut : 0.72 ± 0.07

Nilai presisi relatif rata-rata adalah : 0.1427705

III.DAFTAR PUSTAKA

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